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## Mechanisms for boundary and interior crises of attractors in the quasiperiodically forced Hénon map

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Dynamical transitions associated with sudden changes of the attractor under a small variation of some driving parameter of the system are referred to as the attractor crises [1]. Recently, much attention has been paid to an investigation of the mechanisms for attractor crises in the quasiperiodically forced systems. This interest is conditioned by a special type of dynamical behavior associated with a strange nonchaotic attractor (SNA) [2], which appears in the intermediate region on the border between quasiperiodicity and chaos in such systems.

We study the underlying mechanisms for two types of attractor crises in the quasiperiodically forced Hénon map, which is a representative model for invertible dissipative systems under external two-frequency forcing. A sudden disappearance of the attractor occurs when it collides with an unstable set on the basin boundary. This phenomenon is called the boundary crisis. We show that the boundary crisis may have two different mechanisms in our model system. For small values of the quasiperiodic force amplitude  $\varepsilon$ , a destruction of the chaotic attractor (CA) occurs due to a homoclinic tangency of stable and unstable manifolds of a saddle torus, which lies on the basin boundary. The attractor of the system belongs to a closure of the unstable manifold  $W^U$  of the saddle torus, while the stable manifold  $W^S$  forms its basin boundary. At the moment of homoclinic tangency, trajectories on the attractor approach the saddle torus infinitely closely, and the attractor transforms into a chaotic saddle. However, for large values of  $\varepsilon$ , the mechanism of crisis changes. Instead of destruction of an attractor, homoclinic tangency of  $W^S$  and  $W^U$  causes metamorphosis of its basin, which becomes fractal-like [1,3]. We observe a smooth torus, SNA, and CA with the fractal-like basin of attraction. Using the rational approximations (RAs), we localize a special type of saddle invariant set, which lies on the basin boundary, and is called the "ring-shaped unstable set" (RUS) [4] in accordance with its topology. We show that a collision

of an attractor (torus, SNA, CA) with RUS on the boundary of the basin causes crisis, which destroys the attractor.

The second type of attractor crisis occurs when an attractor undergoes sudden widening due to a collision with an unstable orbit, which lies in the interior of the basin. This transition is called the interior "widening" crisis. We show that a collision of an attractor (SNA, CA) with the "ring-shaped unstable set" within the smooth basin causes "widening" crisis of the attractor. Note, that the attractor type does not change after such crisis.

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### **Chaotic attractors in dissipative systems with discrete symmetry**

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There is a number of well-known three-dimensional flows with quadratic nonlinearities, which demonstrate chaotic behavior. The most popular among them are Lorenz and Rossler systems. In [1] J.Sprott, using the direct computer search, found 19 different three-dimensional chaotic flows of this type which are more simple in the algebraic sense than two above mentioned systems.

Instead of the Sprott idea of "algebraic simplicity", we exploit some symmetry-related ideas similar to those in our previous papers (see, for example, [2]), devoted to studying regular dynamics of nonlinear systems with discrete symmetries. In [3], we found all three-dimensional chaotic flows with quadratic nonlinearities, which possess one of 32 point groups of crystallographic symmetry. It turns out that chaotic flows with quadratic nonlinearities can be only of such crystallographic symmetry:  $C_1$ ,  $C_s$ ,  $C_2$ ,