Introduction

- Fast Sparsely Synchronized Brain Rhythms
  - Population level: Fast oscillations (e.g., gamma rhythm (30–100Hz) and sharp-wave ripples (100–200Hz))
  - Cellular level: Stochastic and intermittent discharges
  - Associated with diverse cognitive functions (e.g., sensory perception, feature integration, selective attention, and memory formation and consolidation)

- Modular Architecture of Real Brain
  - Modular structure of brain: The mammalian brain (e.g., cat and macaque) has been revealed to have a modular structure composed of sparsely linked clusters with spatial localization.
  - Complex topology in modules: Connection architecture of the real brain reveals complex topology such as small-worldliness and scale-freeness which are neither regular nor random.

- Our neuronal model: Clustered Small-World Network (SWN)

- Purpose of Our Study
  - Investigation of Effect of Inter-Modular Synaptic Connections on Emergence of Sparsely Synchronized Brain Rhythms in Clustered SWN

Clustered Small-World Networks

- Clustered Networks with M Small-World (SW) Sub-Networks
  - Intra-modular connection: Small-World Network
  - Inter-modular connection: Random

- M: No. of Sub-Networks (M=3)
- Each Cluster: Small-World Sub-Network
  - Composed of L Inhibitory Fast Spiking (FS) Izhikevich Interneurons

- Izhikevich Interneurons: not only biologically plausible (Hopfian-Huxley neuron-like), but also computationally efficient (IF neuron-like)

Fast Sparsely Synchronized Rhythms in SW Sub-Networks

- Intra-Modular Dynamics
  - Fast Sparsely Synchronized State with the population frequency $f_p=147$ Hz and the individual neuron's mean firing rate $f_I=3.3$ Hz.

Modular and Global Synchronization in Clustered SWN

- Instantaneous Population Spike Rate
  - Instantaneous sub-population spike rate for the $i$th sub-network:
  $$R_{i}(t) = \frac{1}{t_{ex}} \sum_{i=1}^{n} \delta(t - t_{i})$$
  - $\delta(t)$: Spiking rate of the $i$th neuron in the $i$th sub-network
  - $\sum_{i=1}^{n}$: Total number of spikes for the $i$th neuron in the $i$th sub-network

- Gaussian kernel function of band width $h$:
  $$K_{G}(h) = \frac{1}{\sqrt{2\pi h}} e^{-\frac{t^2}{2h^2}}$$

- Instantaneous whole-population spike rate for the whole network:
  $$R_{w}(t) = \frac{1}{t_{ex}} \sum_{i}^{n} \delta(t - t_{i})$$

- State Diagram in the $J_{syn}$-$M$ Plane
  - Modular Synchronization: Mismatching between intra-modular dynamics of sub-networks
  - Global Synchronization: Matching between intra-modular dynamics of sub-networks

Synchronization-Unsynchronization Transition (Route I with $m_{sync}=20$)

- Realistic Thermodynamic Order Parameter
  - Sub-population order parameter for the $i$th sub-network:
  $$Q_{i}(t) = \frac{1}{N_{i}} \sum_{i=1}^{N_{i}} \delta(t - t_{i})$$
  - $Q_{i}(t)$: Order parameter for the $i$th sub-network
  - $N_{i}$: Number of neurons in the $i$th sub-network

- For the synchronized (unsynchronized) state, $Q_{i}$ and $Q_{w}$ approach non-zero (zero) limit values in the thermodynamic limit of $N_{i}$.

- When passing the threshold $E_{crit}(\delta T)$, a transition to unsynchronization occurs.

Characterization of Sync. and Unsync. Using the Spatial Cross-correlation (Route I with $M_{sync}=20$)

- Spatial Cross-Correlation
  - Instantaneous individual spike rate of the $i$th neuron in the $i$th sub-network:
  $$C_{i}(i) = \sum_{j=1}^{N_{i}} (i-j)$$

- Spatial cross-correlation function between the instantaneous individual spike rates of the $(i,j)$ neuronal pair in the $i$th sub-network:
  $$C_{ij}(\tau) = \frac{1}{t_{ex}} \sum_{i,j} (i-j)$$

- For synchronized state: $c_{i,j}$ nearly non-zero constant for whole range of $\tau$
  - Correlation length $L_{c}$

- For unsynchronized state: $c_{i,j}$ nearly zero for whole range of $\tau$:
  - Correlation length $L_{c}$

Characterization of Degree of Synchronization (Route I with $m_{sync}=20$)

- Realistic Statistical-Mechanical Spiking Measure
  - Occupation degree: representing the density of spiking in the raster plot
  - Pacing degree: representing the synchronizing of spiking in the raster plot (average contribution of all microscopic spiking in the stripe)

- With increasing $J_{syn}$:
  - $\langle \xi \rangle$: decreases monotonically.
  - $\langle \eta \rangle$: exhibits the bell-shaped curve.

- Spatial Cross-Correlation Based Measure
  - Subpopulation spatial cross-correlation degree $\langle \xi \rangle$: Similar bell-shaped curve
  - The statistical-mechanical pacing degree between spikes seems to be associated with the spatial cross-correlation degree between neuronal pairs.

Summary

- Investigation of The Effect of Inter-Modular Connections on Emergence of Sparsely Synchronized Cortical Rhythms
  - Occurrence of Modular Sparse Synchronization and Global Sparse Synchronization
  - Depending on the Values of $J_{syn}$ and $m_{sync}$

- Dual Roles of Inter-Modular Coupling Strength $J_{syn}$:
  - For large $J_{syn}$: Destructive role to "spoil" the pacing between sparse spiking
  - For small $J_{syn}$: Constructive role to "favor" the pacing between spiking in each sub-network.

- Role of Number of Inter-Modular Connections $M_{sync}$:
  - Important implications for the role of the Brain Plasticity, which refers to the brain's ability to change its structure and function by modifying the strength or efficacy of synaptic transmission.