Modular and Global Sparse Synchronization in Clustered Small-World Networks of Inhibitory Fast Spiking Izhikevich Interneurons

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Behaving Brain Rhythms via Sparse Synchronization

- **Fast Sparsely Synchronized Brain Rhythms**
  - Population Level: Fast Oscillations
e.g., gamma rhythm (30~100Hz) and sharp-wave ripple (100~200Hz)
  - Cellular Level: Stochastic and Intermittent Discharges
  - Associated with Diverse Cognitive Functions
e.g., sensory perception, feature integration, selective attention, and memory formation and consolidation

- **Sparsely Synchronized Brain Rhythms**
  
  Individual Neurons: Intermittent and Stochastic Firings like Geiger Counters
  Small-Amplitude Fast Population Rhythm via Sparse Synchronization of Individual Complex Firings
  
  Coupled oscillators model: Inappropriate for investigation of the sparsely synchronized rhythms
  → **Coupled Subthreshold and/or Suprathreshold Neurons in the Presence of Strong Noise**
  They exhibit noise-induced complex firing patterns

Gamma rhythm in visual cortex of behaving monkey

Sparsely Synchronized Brain Rhythms

Individual Neurons: Intermittent and Stochastic Firings like Geiger Counters
Small-Amplitude Fast Population Rhythm via Sparse Synchronization of Individual Complex Firings

Coupled oscillators model: Inappropriate for investigation of the sparsely synchronized rhythms

→ **Coupled Subthreshold and/or Suprathreshold Neurons in the Presence of Strong Noise**
They exhibit noise-induced complex firing patterns
• Complex Brain Network
Connection architecture of the real brain reveals complex topology such as small-worldness and scale-freeness which are neither regular nor random.

• Modular Structure of Brain
The mammalian brain (e.g., cat and macaque) has been revealed to have a modular structure composed of sparsely linked clustered with spatial localization.
Brain Plasticity – Learning and Memory

● Brain Plasticity

Brain plasticity refers to the brain’s ability to change throughout life.

Brain plasticity occurs in the brain:
  At the beginning of life, In case of brain injury, and
  Through adulthood (whenever something new is learned and memorized)

● Synaptic Plasticity

Synaptic plasticity is the ability of a synapse between neurons to change in strength over time.
  → Change in the Strength of Synaptic Connections

● Non-Synaptic Plasticity

Non-synaptic plasticity involves modification of ion channel function in the axon, dendrites, and cell body.
  → Change in the Synaptic Path Ways
Sparse Synchronization in Clustered Small-World Networks

- **Clustered Small-World Network**
  
  Intra-modular connection: Small-World Network
  Inter-modular connection: Random

- **Interneuronal Network (I-I Loop)**
  
  Playing the role of the backbones of many brain rhythms by providing a synchronous oscillatory output to the principal cells

- **FS Izhikevich Interneuron**
  
  Izhikevich Interneuron Model: not only biologically plausible (Hodgkin-Huxley neuron-like), but also computationally efficient (IF neuron-like)

- **Effect of Inter-Modular Synaptic Connections**
  
  Effect of Inter-Modular Synaptic Connection on Sparsely-Synchronized Brain Rhythms in Clustered Small-World Network
  ⇒ Implications for The Role of The Brain Plasticity
Fast Sparsely Synchronized Rhythms in Small-World Network

- **Intra-Modular Dynamics**

Fast Sparsely Synchronized State with the population frequency $f_p = 147$ Hz and the individual neuron’s mean firing rate $f_i = 33$ Hz.

\[ I_{DC} = 1500, M_{syn}^{(intra)} = 50, J_{intra} = 1400, D = 500, p_{rewiring} = 0.25, L = 10^3 \]
Modular, Global Synchronization and Unsynchronization in Clustered Small-World Network

- **Instantaneous Population Spike Rate**

  Instantaneous sub-population spike rates for the \( i \)th sub-network:
  \[
  R_s^{(i)}(t) = \frac{1}{L} \sum_{i=1}^{L} \sum_{j=1}^{n(t)} K_h(t - t_j^{(iL)})
  \]

  Gaussian kernel function of band width \( h \):
  \[
  K_h(t) = \frac{1}{\sqrt{2\pi h}} e^{-t^2/2h^2}, \quad -\infty < t < \infty
  \]

  Instantaneous whole-population spike rates for the whole network:
  \[
  R_w(t) = \frac{1}{M} \sum_{i=1}^{M} R_s^{(i)}(t) = \frac{1}{M \cdot L} \sum_{i=1}^{M} \sum_{j=1}^{L} \sum_{i}^{n(t)} K_h(t - t_j^{(iL)})
  \]

- **State Diagram in the \( J_{\text{inter}} - M_{\text{syn}} \) Plane**

  Modular Synchronization: Mismatching between modular synchronization of sub-networks.
  Global Synchronization: Matching between modular synchronization of sub-networks
  Unsynchronization

  \[ M = 3, M_{\text{syn}}^{(\text{inter})} = 20 \]
Synchronization-Unsynchronization Transition
Effect of Large Inter-modal Synaptic Strength

- Realistic Thermodynamics Order Parameter
  Sub-population order parameter for the $i$th sub-network:
  \[
  \mathcal{O}_s^{(i)} \equiv (R^{(i)}_s(t) - R_{s}^{(i)}(t))^2
  \]
  Whole-population order parameter for the whole network:
  \[
  \mathcal{O}_w = (V_w(t) - V_{w}(t))^2
  \]
  For the synchronized state, $\mathcal{O}_s^{(i)}$ and $\mathcal{O}_w$ approach a non-zero limit value for $N \to \infty$.
  For the unsynchronized state, $\mathcal{O}_s^{(i)}$ and $\mathcal{O}_w$ approach a zero limit value for $N \to \infty$. 

\[
J_{\text{inter}} = 1600, \quad L = 10^3
\]
\[
J_{\text{inter}} = 1600, \quad L = 10^4
\]
\[
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\]
\[
J_{\text{inter}} = 1700, \quad L = 10^4
\]
Characterization of Synchronization and Unsynchronization Using the Spatial Cross-correlation

**Spatial Cross-Correlation**

Instantaneous individual spike rate: \( r_i^{(t)}(t) = \sum_{j=1}^{n_i^{(t)}} K_{ij}(t-t_j^{(t)}) \)

Normalized temporal cross-correlation function between in the instantaneous individual spike rate in the \( \mathcal{A} \)th sub-network sub-population spike rates:

\[
C_{i,j}^{(t)}(\tau) = \frac{\Delta r_i^{(t)}(t+\tau) \cdot \Delta r_j^{(t)}(t)}{\sqrt{\Delta r_i^{(t)}(t)^2} \sqrt{\Delta r_j^{(t)}(t)^2}}
\]

Spatial cross-correlation function: \( C_{i}^{(t)} = \frac{1}{L} \sum_{i=1}^{L} C_{i,i+t}^{(t)}(0) \) for \( L = 1, ..., L/2 \).

For synchronized state:
- \( C_{i}^{(t)} \): nearly non-zero constant for whole range of \( l \).
- Correlation length = \( L/2 \)

For unsynchronized state:
- \( C_{i}^{(t)} \): nearly zero for whole range of \( l \).
- Correlation length = 0
Modular-Global Synchronization Transition
Effect of Small Inter-modular Synaptic Strength

- Modular and Global Synchronization

Normalized temporal cross-correlation function between the instantaneous sub-population spike rates:

\[ C_{I,J}(\tau) = \frac{\Delta R_S^{(I)}(t + \tau) \cdot \Delta R_S^{(J)}(t)}{\sqrt{\Delta R_S^{(I)}(t)^2} \sqrt{\Delta R_S^{(J)}(t)^2}} \]

\[ \Delta R_S^{(I)}(t) = R_S^{(I)}(t) - \bar{R}_S^{(I)}(t) \]

- Cross-Correlation Modularity Measure

\[ C_M = \frac{2}{M(M-1)} \sum_{I=1}^{M} \sum_{J=I+1}^{M} C_{I,J}(0) \]

- Cross-Correlation Modularity Measure

\[ J_{\text{inter}} < J_{\text{inter}}^{**} (\approx 268) : \text{Modular Sync.} : 0 < \langle C_M \rangle_r < 1 \]

\[ J_{\text{inter}}^{**} < J_{\text{inter}} < J_{\text{inter}}^* : \text{Global Sync.} : \langle C_M \rangle_r \approx 1 \]
Characterization of Degree of Synchronization

- **Realistic Statistical-Mechanical Spiking Measure**

  Occupation degree: representing the density of stripe in the raster plot
  Pacing degree: representing the smearing of stripe in the raster plot (average contribution of all microscopic spikes in the stripe)

  With increasing $J_{\text{inter}}$
  \[
  \left\langle O_S^{(I)} \right\rangle_r \text{ decreases monotonically.}
  \]
  \[
  \left\langle P_S^{(I)} \right\rangle_r \text{ exhibits the bell-shaped curve.}
  \]

  Modular Sync. : $\left\langle O_S^{(I)} \right\rangle_r < \left\langle O_W \right\rangle_r$, $\left\langle P_S^{(I)} \right\rangle_r < \left\langle P_W \right\rangle_r$, $\left\langle M_S^{(I)} \right\rangle_r < \left\langle M_S^{(W)} \right\rangle_r$
  Global Sync. : $\left\langle O_S^{(I)} \right\rangle_r \geq \left\langle O_W \right\rangle_r$, $\left\langle P_S^{(I)} \right\rangle_r \geq \left\langle P_W \right\rangle_r$, $\left\langle M_S^{(I)} \right\rangle_r \geq \left\langle M_S^{(W)} \right\rangle_r$

- **Spatial Cross-Correlation Based Measure**

  Average value of spatial cross-correlation function: Same behavior with average pacing degree.
  $\Rightarrow$ Implication with the measure for the degree of synchronization

  $M = 3, M_{\text{syn}}^{(\text{inter})} = 20$
Summary

• Investigation of The Effect of Inter-Connection on Emergence of Sparsely Synchronized Cortical Rhythms

Occurrence of Modular Sparse Synchronization and Global Sparse Synchronization

Modular sparse synchronization: the population behavior reveals the clustering structure due to some mismatching between the intra-modular dynamics of the sub-networks

Global Sparse Synchronization: the population behavior is globally identical, independently of the cluster structure, because of the perfect matching between the intra-modular dynamics of sub-networks

Dual Roles of Inter-Modular Coupling Strength $J_{\text{inter}}$ Depending on Its Strength:

For large $J_{\text{inter}}$ → Destructive role to “spoil” the pacing between sparse spikings

For small $J_{\text{inter}}$ → Constructive role to “favor” the pacing between spikings in each sub-network.

Role of Number of Inter-Modular Connection Probability:

Constructive role to “favor” global communication between sub-networks

Important implications for the role of the Brain Plasticity which refers to the brain’s ability to change its structure and function by modifying the strength or efficacy of synaptic transmission.