Noise-Induced Burst and Spike Synchronizations in An Inhibitory Small-World Network of Subthreshold Bursting Neurons

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- **Burstings with the Slow and Fast Time Scales**
  Bursting: Neuronal activity alternates, on a *slow timescale*, between a silent phase and an active (bursting) phase of *fast repetitive spikings*

  Representative examples of bursting neurons: chattering neurons in the cortex, thalamocortical relay neurons, thalamic reticular neurons, hippocampal pyramidal neurons, Purkinje cells in the cerebellum, pancreatic β-cells, and respiratory neurons in pre-Botzinger complex

- **Complex Brain Network**
  Network Topology: Complex (Neither Regular Nor Random)

  **Effect of Network Architecture on Burst and Spike Synchronization**
Burst and Spike Synchronizations of Bursting Neurons

- **Synchronization of Bursting Neurons**
  Two Different Synchronization Patterns Due to the **Slow and Fast Time Scales** of Bursting Activity

- **Burst Synchronization**
  **Synchrony on the Slow Bursting Timescale**
  Temporal coherence between the active phase (bursting) onset or offset times of bursting neurons

- **(Intraburst) Spike Synchronization**
  **Synchrony on the Fast Spiking Timescale**
  Temporal coherence between intraburst spikes fired by bursting neurons in their respective active phases
Small-World Network of Inhibitory Bursting Hindmarsh-Rose Neurons

- Small-World Network of Inhibitory Bursting Hindmarsh-Rose Neurons

Start with directed regular ring lattice with \( N \) neurons where each neuron is coupled to its first \( k \) neighbors.

Rewire each outward connection at random with probability \( p \) such that self-connections and duplicate connections are excluded.

\[
\begin{align*}
\frac{dx_i}{dt} &= y_i - ax_i^3 + bx_i^2 - z_i + I_{DC} + D \xi_i - I_{\text{syn},i}, \\
\frac{dy_i}{dt} &= c - dx_i^2 - y_i, \\
\frac{dz_i}{dt} &= r[s(x_i - x_o) - z_i],
\end{align*}
\]

- Parameters in the single HR neuron

\( a = 1, \ b = 3, \ c = 1, \ d = 5, \ r = 0.001, \ s = 4, \ x_o = -1.6 \)

- Parameters for the synaptic current

\( X_{\text{syn}} = -2, \ x^*_s = 0, \ \delta = 30, \ \alpha = 10 \text{ ms}^{-1}, \ \beta = 0.1 \text{ ms}^{-1} \)

- Bursting Activity of the Single HR Neuron

\[
I_{\text{syn},i} = \frac{J}{N-1} \sum_{j(i)}^N w_{i,j} g_j(t)(x_i - X_{\text{syn}}), \quad g(x_i) = 1/[1+e^{-(x-x_i)\delta}].
\]
Synchronization in Random Neural Network

- State Diagram

  Complete Synchronization:
  Including both the burst and spike synchronizations

  Bursting Synchronization:
  Without spike synchronization

  Complete Unsynchronization

- Instantaneous Population Firing Rate Kernel Estimate

  Instantaneous population firing rates:
  \[ R_s(t) = \frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{n_i} K_h(t - t_j^{(i)}) \]

  Gaussian kernel function of band width \( h \):
  \[ K_h(t) = \frac{1}{\sqrt{2\pi h}} e^{-t^2/2h^2}, \quad -\infty < t < \infty \]
Burst and Spike Synchronization

- **Emergence of Burst Synchronization for J=0.35 (Route A)**

  Unsynchronization (D=0.02)
  ↓
  Burst Synchronization
  Bursting bands in the raster plot
  Slow oscillation in $R(t)$
  ↓
  Unsynchronization (D=0.11)

- **Emergence of Burst & Spike Synchronizations for J=0.6 (Route B)**

  Unsynchronization (D=0.015)
  ↓
  Burst Synchronization
  ↓
  Complete Synchronization (D=0.03)
  Both burst & spike synchronizations
  ↓
  Burst Synchronization
  ↓
  Unsynchronization (D=0.14)
With decreasing the rewiring probability, the region of burst synchronization decreases slowly, while the region of spike synchronization shrink rapidly.
Characterization of Burst and Spike Synchronizations via Separation of the Slow (Bursting) and Fast (Spiking) Time Scales

Raster Plot of Neural Spikes:
Population synchronization may be well visualized. Obtained in experiments

Instantaneous Population Firing Rate (IPFR) $R$
Describing the population behaviors

Separation of the Slow and Fast Timescale of Bursting Activity via Frequency Filtering

Instantaneous Population Bursting Rate (IPBR) $R_b$
Describing the Slow Bursting Behavior

Instantaneous Population Spiking Rate (IPSR) $R_s$
Describing the Fast Spiking Behavior

Thermodynamic Intraburst Spiking Order Parameter
Characterization of Intraburst Spiking Transition

Statistical-Mechanical Intraburst Spiking Measure
Characterization of Degree of Intraburst Spike Sync.
Characterization of Bursting Transition Based on $R_b$

- Investigation of Bursting States via Separation of Slow Timescale

Separation of the slow timescale from IPFR $R$ via low-pass filtering ($f_c=10\text{Hz}$) $\rightarrow$ IPBR $R_b$

- Thermodynamic Bursting Order Parameter

$$\mathcal{O}_b \equiv (R_b(t) - \overline{R_b(t)})^2$$

Unsynchronized Bursting State: $N \to \infty$, then $\mathcal{O}_b \to 0$
Synchronized Bursting State:
$N \to \infty$, then $\mathcal{O}_b \to$ Non-zero value
Characterization of Burst Synchronization Based on Bursting Onset and Offset Times

Raster Plot of Neural Spikes

Raster Plots of Active Phase (Bursting) Onset or Offset Times

More direct visualization of bursting behavior

Instantaneous Population Bursting Rates (IPBRs) for Active Phase Onset and Offset Times

Thermodynamic Bursting Order Parameter
Characterization of Bursting Transition

Statistical-Mechanical Bursting Measure
Characterization of Degree of Burst Sync.
Characterization of Bursting Transition Based on $R_{b}^{(on)}$ and $R_{b}^{(off)}$

- Investigation of Bursting States Based on Bursting Onset and Offset Times

Another Raster plots of bursting onset and offset times and smooth IPBR kernel estimates $R_{b}^{(on)}$ and $R_{b}^{(off)}$.

- Thermodynamic Bursting Order Parameters Based on $R_{b}^{(on)}$ and $R_{b}^{(off)}$

\[
\mathcal{O}_{b}^{(on)} \equiv (R_{b}^{(on)}(t) - \overline{R_{b}^{(on)}})^2
\]

\[
\mathcal{O}_{b}^{(off)} \equiv (R_{b}^{(off)}(t) - \overline{R_{b}^{(off)}})^2
\]
Statistical-Mechanical Bursting Measure Based on $R_{b}^{(on)}$ and $R_{b}^{(off)}$

- **Bursting measure** of $i$th bursting stripe in the raster plot times
  \[ M_{i}^{(b, on)} = O_{i}^{(b, on)} \cdot P_{i}^{(b, on)} \]

- **Occupation degree** of bursting onset times
  \[ O_{i}^{(b, on)} = \frac{N_{i}^{(b)}}{N} \quad N_{i}^{(b)}: \text{No. of bursting HR neurons in the } i^{th} \text{ bursting stripe} \]

- **Pacing degree** of bursting onset times: contribution of bursting onset times to the macroscopic IPBR $R_{b}^{(on)}$
  \[ P_{i}^{(b, on)} = \frac{1}{B_{i}^{(on)}} \sum_{k=1}^{B_{i}^{(on)}} \cos \Phi_{k} \]
  \[ \Phi_{k}: \text{Global bursting phase based on IPBR } R_{b}^{(on)} \]
  \[ B_{i}^{(on)}: \text{Total No. of microscopic bursting onset times in the } i^{th} \text{ bursting stripe} \]

- **Statistical-Mechanical Bursting Measure**
  Based on IPBR $R_{b}^{(on)}$
  \[ M_{b}^{(on)} = \frac{1}{N_{b}} \sum_{i=1}^{N_{b}} M_{i}^{(b, on)} \]

- **Statistical-Mechanical Bursting Measure**
  \[ M_{b} = [M_{b}^{(on)} + M_{b}^{(off)}] / 2 \]
Characterization of Intraburst Spiking Transition Based on $R_s$

- **Investigation of Intraburst Spiking States via Separation of Fast Time Scale**
  - IPSR $R_s$ via band-pass filtering [lower and higher cut-off frequencies of 30Hz (high-pass filter) and 90 Hz (low-pass filter)]

\[ \text{Thermodynamic Intraburst Spiking Order Parameter} \]

Mean square deviation of $R_s$ in the $i$th global bursting cycle: \[ \mathcal{O}_s^{(i)} = (R_s(t) - \overline{R_s}(t))^2 \]

Thermodynamic spiking order parameter:

\[ \mathcal{O}_s = \frac{1}{N_b} \sum_{i=1}^{N_b} \mathcal{O}_s^{(i)} \]

Unsynchronized Spiking State: $N \to \infty$, then $\mathcal{O}_s \to 0$

Synchronized Spiking State:

$N \to \infty$, then $\mathcal{O}_s \to$ Non-zero value
Intraburst Spiking Measure Based on IPSR $R_s$

**Spiking measure** of jth global spiking cycle in the ith bursting cycle

$$M_{i,j}^{(s)} = O_{i,j}^{(s)} \cdot P_{i,j}^{(s)}$$

**Occupation degree** of spiking times

$$O_{i,j}^{(s)} = \frac{N_{i,j}^{(s)}}{N}$$

$N_{i,j}^{(s)}$: No. of spiking HR neurons in the jth spiking cycle and the ith bursting cycle

**Pacing degree** of spiking times:

Contribution of spiking times to the macroscopic IPSR $R_s$

$$P_{i,j}^{(s)} = \frac{1}{N_{i,j}^{(s)}} \sum_{k=1}^{N_{i,j}^{(s)}} \cos \Phi_k$$

$N_{i,j}^{(s)}$: Total No. of microscopic spiking times

**Statistical-Mechanical Spiking Measure**

Based on IPSR $R_s$

$$M_i^{(s)} = \frac{1}{N_s} \sum_{j=1}^{N_s} M_{i,j}^{(s)}$$

**Statistical-Mechanical Spiking Measure**

$$M_s = \frac{1}{N_b} \sum_{i=1}^{N_b} M_i^{(s)}$$
Summary

• Investigation of The Effect of Network Architecture on the Burst and Spike Synchronization

  Occurrence Burst and Spike Synchronization in Random Network

  Investigation of Burst and Spike Synchronization with Decreasing the Rewiring Probability
   Burst synchronization decreases slowly, while spike synchronization shrink rapidly.

  Characterization of Burst and Spike Synchronization by Using the Realistic Thermodynamic Bursting and Spiking Order Parameter Based on the IPBR and IPSR Kernel Estimates through Frequency Filtering.

  Characterization of Degree of Burst and Spike Synchronization by Using the Realistic Statistical-Mechanical Bursting and Spiking Measure Based on the IPBR and IPSR Kernel Estimates through Frequency Filtering.