Fast Sparsely Synchronized Brain Rhythms in A Scale-Free Neural Network[1]

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BACKGROUND

- **Fast Sparsely Synchronized Brain Rhythms**
  - Population level: Fast synchronous oscillations
    [e.g. gamma rhythm (30~100Hz) and sharp-wave ripple (100~200Hz)]
  - Cellular level: Stochastic and intermittent discharges
  - Associated with diverse cognitive functions [e.g., sensory perception, feature integration, selective attention, and memory formation and consolidation]
  - Previous works of Brunel et al.[2]: Developed a framework appropriate for fast sparse synchronization in global and random networks. But, realistic brain networks: neither regular nor random

- **Scale-Free Networks (SFNs)**
  - Scale-Free Structure of Real Brain: Rat hippocampal networks and human brain functional networks have been revealed to show power-law degree distribution (i.e., scale-free property)
  - SFNs: Inhomogeneous networks with a few "Hubs" (i.e., supernodes)
    cf., Random graphs and Small-World networks: Statistically homogeneous networks

SPECIFIC AIMS

- To Investigate Emergence of Sparsely Synchronized Brain Rhythms in SFNs by Varying J (Synaptic Inhibition Strength) and D (Noise Intensity)

- To Study The Effect of Network Architecture on Sparse Synchronization by Varying (a) the degree of symmetric attachment, (b) the degree of asymmetric attachment, and (c) the degree of attachment between pre-existing nodes

METHODS

Scale-Free Networks

- **Evolved via two independent α- and β-processes:**
  - **α-process**
    Corresponding to a directed version of the Barabási-Albert (B-A) model[3] (i.e., growth and preferential directed attachment)
  - **β-process**
   Preferential attachment between pre-existing nodes without adding new nodes
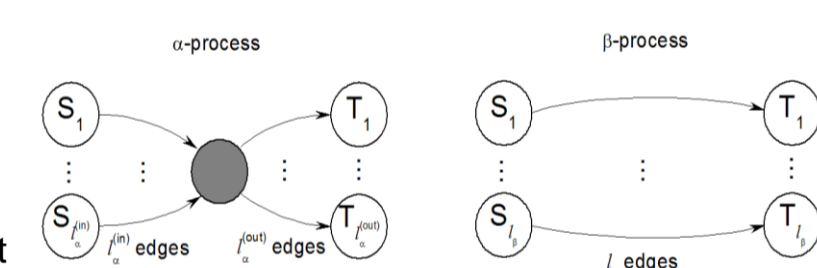
- **Composed of N Inhibitory Izhikevish Fast Spiking Interneurons[4]**

RESULTS

1. State Diagram in the J-D Plane
[1-4:The Directed B-A SFN with Symmetric Preferential Attachment ($I_{DC} = 1500$, $l_\alpha^{(in)} = l_\alpha^{(out)} \equiv l_\alpha = 25$)]

Full Synchronization:
Population frequency f_p
= Mean firing rate (MFR) f_i of individual neurons
Partial Synchronization: $f_p > <f_i>$
Sparse Synchronization: $f_p > 4<f_i>$
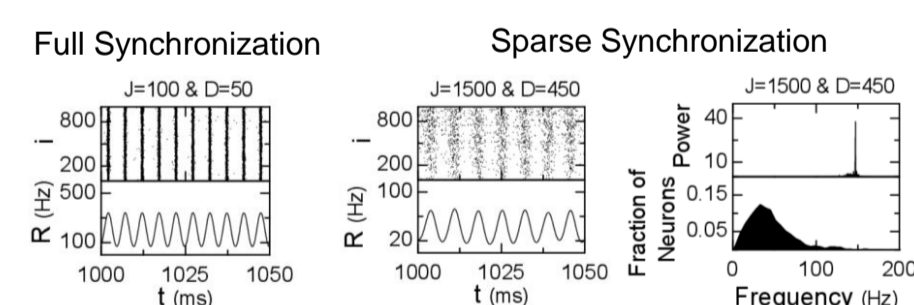
For $J > 173$, the full synchronization is developed into partial synchronization, and then the partial synchronization evolves into sparse synchronization for $J > 1440$.
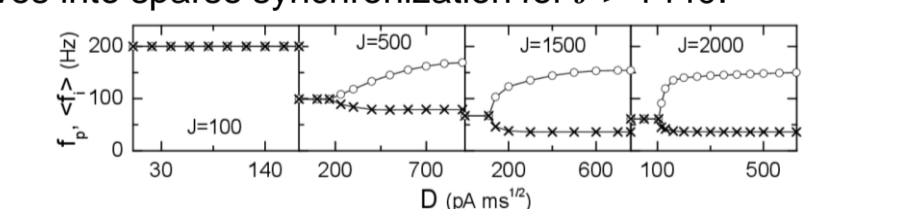
2. Fast Sparse Synchronization
(2-4: $J = 1500$)

- **Instantaneous Population Spike Rate (IPSR)**
  - IPSR $R(t)$: Obtained via convolution (or blurring) of each spike in the raster plot with the Gaussian kernel function
  - Full Synchronization
    Clear stripes in the raster plot of spikes and regular oscillating $R(t)$
    Single peak in the interspike interval histogram (ISIH)
  - Partial & Sparse Synchronization
   Sparse stripes in the raster plot and Multiple peaks in the ISIH → Stochastic spike skipping
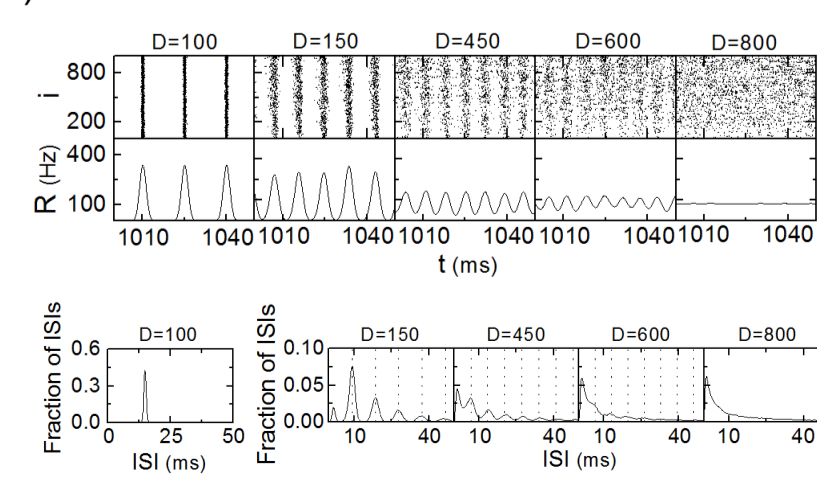
RESULTS (Continued)

3. Synchronization-Unsynchronization Transition

- **Realistic Thermodynamic Order Parameter**
  Realistic thermodynamics order parameter O:
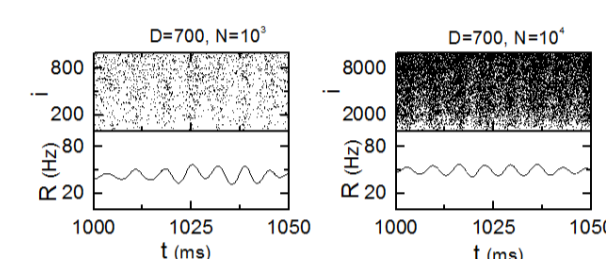 Representing the time-averaged fluctuation of the IPSR $R(t)$

  For the synchronized (unsynchronized) state, O approaches non-zero (zero) limit values in the thermodynamic limit of $N \to \infty$.
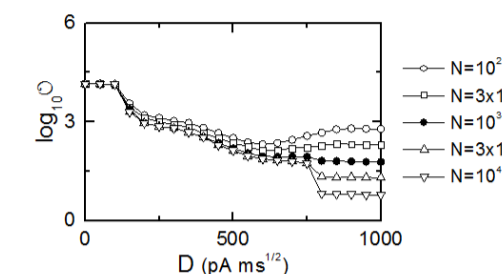
  When passing the threshold D^* ($\simeq$759), a transition to unsynchronization occurs.
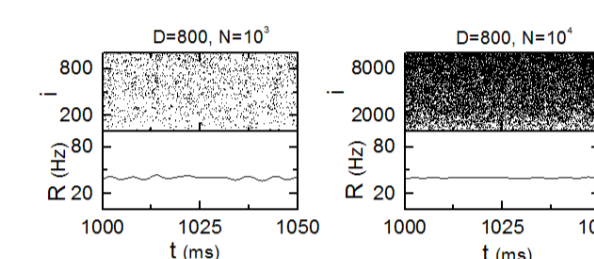

- **Synchronized State**
  Sparse stripes are formed in the raster plot
  $R(t)$ shows regular oscillation

- **Unsynchronized State**
  Sparse spikes are scattered in the raster plot
 $R(t)$ becomes nearly stationary

4. Characterization of Degree of Sparse Synchronization

- **Realistic Statistical-Mechanical Spiking Measure[5]**

  - Statistical-Mechanical Spiking Measure M_s: Given by the product of the occupation and the pacing degrees of spikes in the raster plot.

  - Occupation degree $<O_p>$: representing the average density of stripes in the raster plot
  - Pacing degree $<P_i>$: representing the average smearing of stripes in the raster plot
    (average contribution of all microscopic spikes to the instantaneous population spike rate)

  With increasing D,
  $<O_p>$: drops abruptly from 1 just after the break-up of the full synchronization, and then it saturates to a non-zero limit. $<P_i>$: decreases monotonically to zero after the break-up of the full synchronization.
  $\Rightarrow M_s$: abruptly drops just after the break-up of the full synchronization and then slowly decreases to zero.
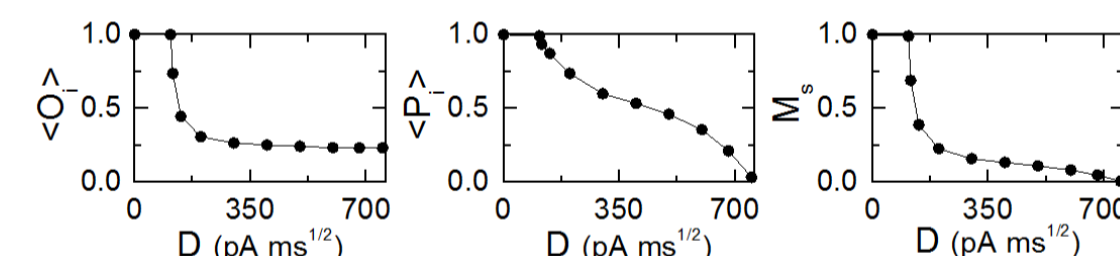
5. Effect of The Symmetric Attachment Degree l_α on Sparse Synchronization

- **Effect of l_α on the Network Topology**
  - Average path length L_p: Typical separation between two nodes in the network

  - Betweenness centrality B_i of the ith node: Potentiality in controlling communication between other nodes
  - Betweenness centralization C_b: Degree to which B_{max} of the head hub exceeds the B_i of all the other nodes

  As l_α is increased, the average in-degrees of the hub and the peripheral groups are increased, which results in the increase in the total number of connections
  → L_p & C_b decrease.

  $\Rightarrow$ Efficiency of communication between nodes becomes better, which may lead to increase in the degree of sparse synchronization.
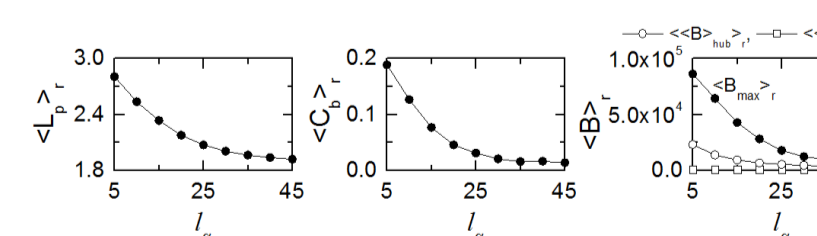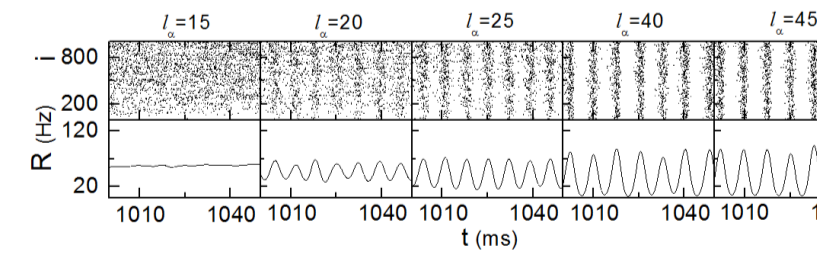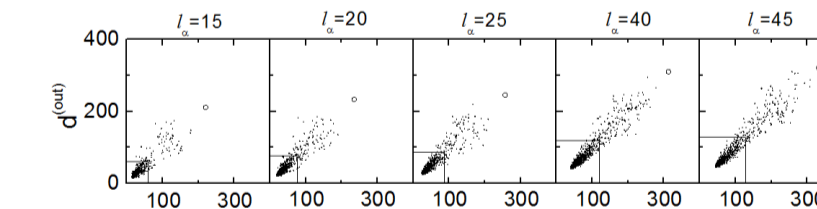
- **Effect of l_α on the Population Dynamics**
  As l_α is increased, occupation degree increases very slowly and pacing degree increases thanks to the increased number of total connections → Statistical-mechanical spiking measure M_s increases.
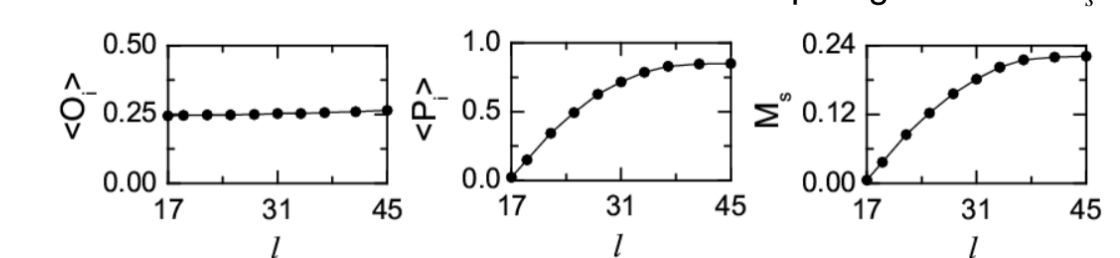
RESULTS (Continued)

6. Effect of The Asymmetric Parameter Δl_α on Sparse Synchronization
($l_\alpha^{(in)} = l_\alpha + \Delta l_\alpha$, $l_\alpha^{(out)} = l_\alpha - \Delta l_\alpha$, and $l_\alpha = 25 \to l_\alpha^{(in)} + l_\alpha^{(out)} = 2l_\alpha =$constant)

- **Effect of Δl_α on the Network Topology**
  As $|\Delta l_\alpha|$ is increased, mismatching between the in- and the out-degrees increases → L_p and C_b increase symmetrically because both inward and outward links are involved equally in computation of L_p and C_b ⇒ Efficiency of communication between nodes becomes worse.

- **Effect of Δl_α on the Individual and Population Dynamics**
  Unlike the symmetric change in L_p and C_b, sparse synchronization varies depending on the sign of Δl_α. As Δl_α is increased, ensemble-averaged MFR decreases slowly and distribution of MFR is much reduced → Occupation degree $<O_p>$ decreases, while Pacing degree $<P_i>$ increases ⇒ Statistical-mechanical spiking measure M_s exhibits the bell-shaped curve.
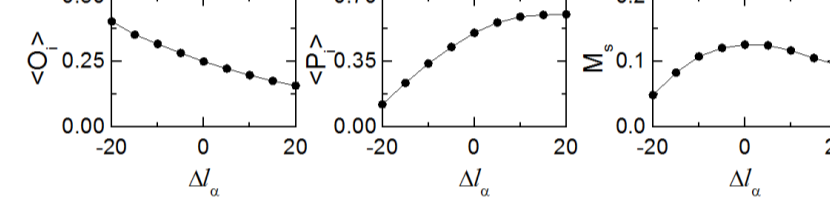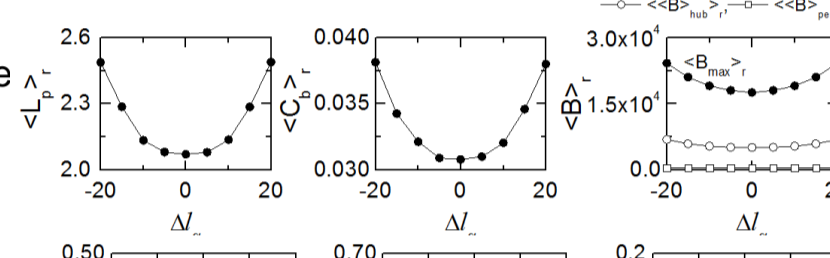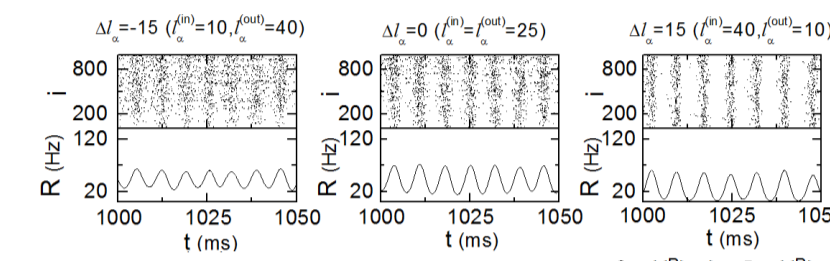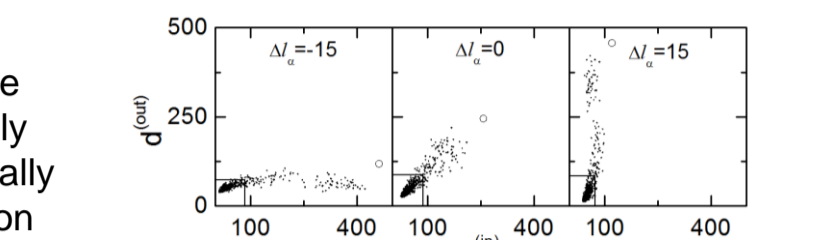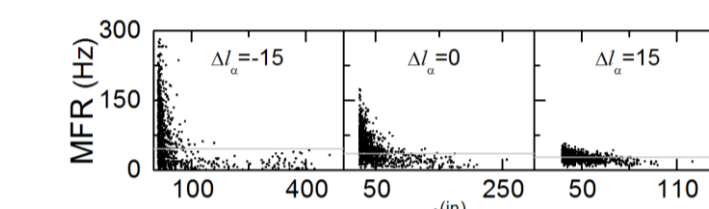

### 7. Effect of β-process on Sparse Synchronization
(β: occurrence probability of the β-process)

- **Effect of β on the Network Topology**
  As β is increased, secondary hub group is intensified, which leads to increase in the total number of connections → L_p & C_b decrease ⇒ Efficiency of communication between nodes becomes better, which may lead to increase in the synchronization degree.

- **Effect of β on the Individual and Population Dynamics**
  As β is increased, ensemble-averaged MFR decreases slightly and distribution of MFR is much reduced → Occupation degree $<O_p>$ decreases very slowly, while Pacing degree $<P_i>$ increases ⇒ Statistical-mechanical spiking measure M_s increases.
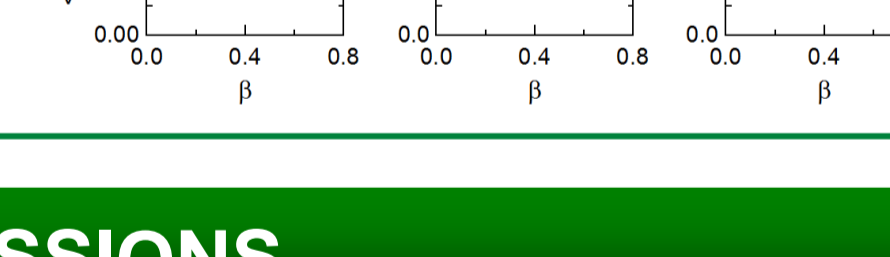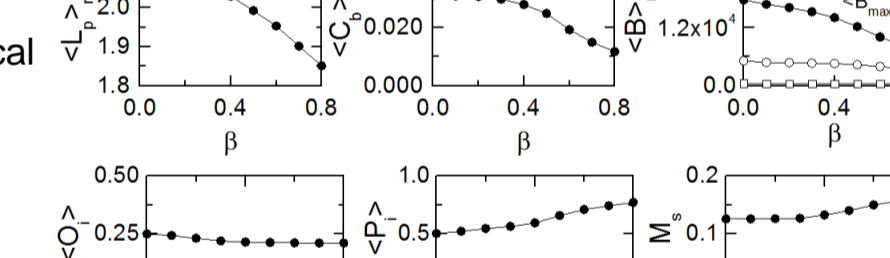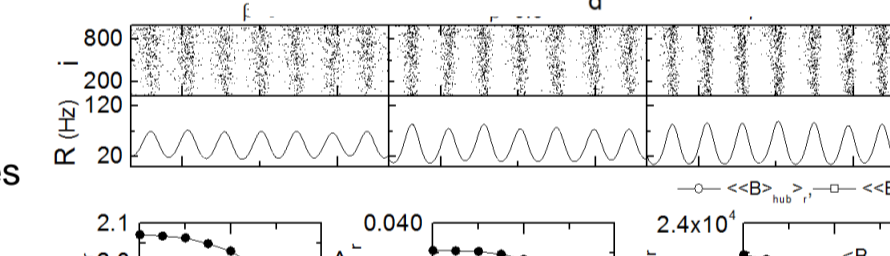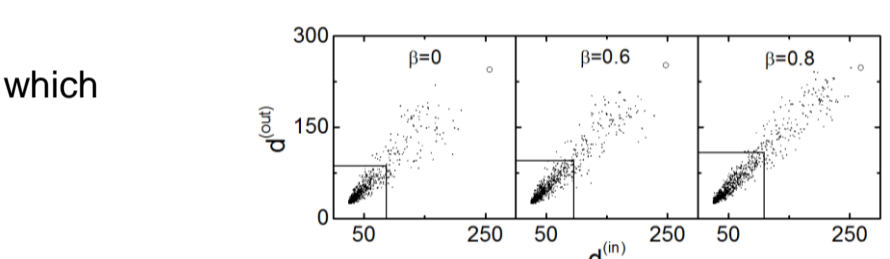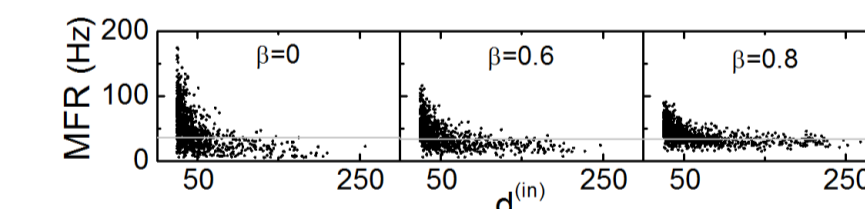
DISCUSSIONS

- **Investigated Emergence of Sparsely Synchronized Rhythms in SFNs**
  Appearance of Sparse Synchronization for large J and large D

- **Sparse Synchronization vs. Full Synchronization**
  For the case of sparse synchronization, contributions of individual dynamics to population synchronization vary depending on degrees (i.e., hub and peripheral groups), unlike the full synchronization → Revealing the inhomogeneous network structure

- **Effect of Network Architecture on Sparse Synchronization**
  Not only L_p and C_b (affecting global communication), but also in-degree distribution (affecting individual dynamics) are important network factors to determine the synchronization degree.
  → A harmony between these network factors is essential for effective synchronization.

REFERENCES

[1] Kim S.-Y. & Lim W. (2015) Fast sparsely synchronized brain rhythms in a scale-free neural network. Phys. Rev. E 92, 022717.
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[3] Albert R. & Barabási A.-L. (2002) Statistical mechanics of complex networks. Rev. Mod. Phys. 74, 47-97.
[4] Izhikevich E.M. (2007) Dynamical Systems in Neuroscience. MIT Press, Cambridge.
[5] Kim S.-Y. & Lim W. (2014) Realistic thermodynamic and statistical-mechanical measures for neural synchronization. J. Neurosci. Methods 226, 161-170.